Workshop on Logical Dialogue Games
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Quick, Quick, Slow: The Foxtrot of Completeness Proofs in Dialogue Logic

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Erstens möchte ich Professor Fermüller, und alle die an der Organisation dieser Tagung beteiligt sind, ganz herzlich danken für die freundliche Einladung. Es gibt mir eine Chance mit Ihnen über meine alte Liebe zu sprechen: die Theorie der logischen Dialogspiele.

Well, I'll continue in English. I'm going to talk to you just a bit about the history of the field from the fifties to the eighties, after that the more serious work of this workshop may start.

But first a word about the title: The word completeness may be taken in two ways senses. Most often a system is said to be complete if it can do all it is supposed to do, but sometimes it also means that the system can do no more. Completeness in this second sense implies soundness. The title is meant in this second sense. I could have used the word "equivalence," but I like "completeness" better.

1 A is complete w.r.t. B in the first sense iff there is a an A for everything for which there is a B (or B holds).

2 A is complete w.r.t. B in the second sense iff there is a an A for everything for which there is a B (or B holds) and vice versa.

Completeness (sense 2) proofs for dialogue systems split into two parts:

a) From winning strategies (B) to deductions (A)
"There is a deduction (A) for any winning strategy (B)"

Here we have completeness (sense 1) of the deductive system (A) w.r.t. the dialogue system (B),
and soundness of the dialogue system (B) w.r.t. the deductive system (A)
(Staunch dialecticians prefer the first reading)

b) From deductions (A) to winning strategies (B)
"There is a winning strategy (B) for any deduction (A)"

Here we have completeness (sense 1) of the dialogue system (B) w.r.t. the deductive system (A),
and soundness of the deductive system (A) w.r.t. the dialogue system (B)
(Here staunch dialecticians prefer the second reading)

Why was there so much fuss about completeness (sense 2) of dialogue logics?

Dialogue logic started in 1958 with Paul Lorenzen's lecture *Logik und Agon* published in 1960, followed the next year by his better known Warsaw lecture *Ein dialogisches Konstruktivitätskriterium,* published in 1961. Both papers were reprinted in the Lorenzen and Lorenz volume of 1978.

It is not that connections with other kinds of logic were not seen by Lorenzen at the time. They were, as is evident from the letter he wrote to Evert Willem Beth just before the Warsaw conference of 1959 where he was going to present the second early paper on the subject. Let us have a look at that letter. First have a look at the handwriting .

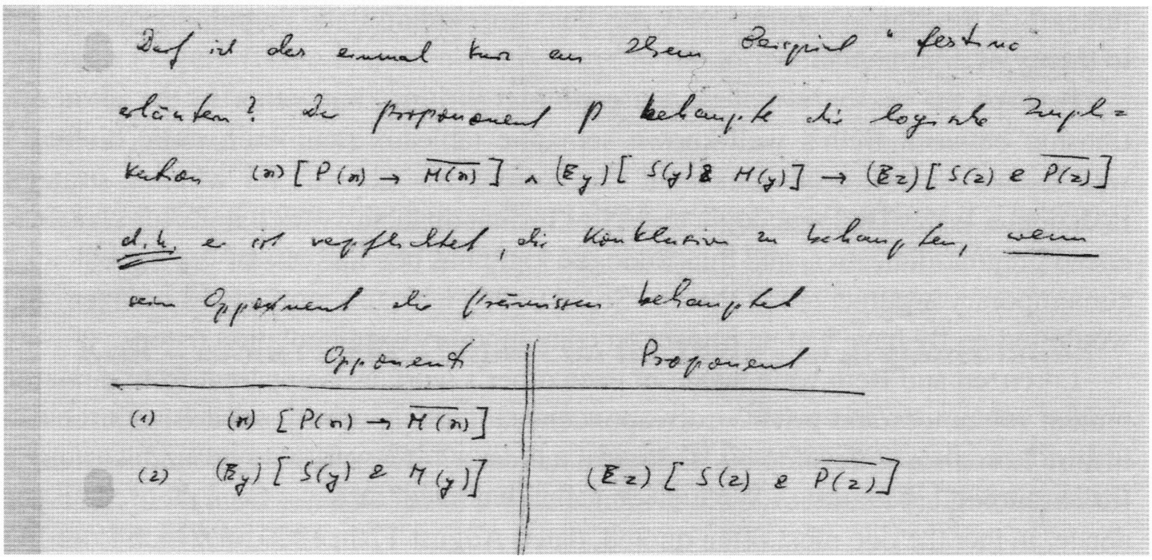
What you are seeing is one of the first dialogical tableaux. Now, here is the whole letter, retyped by me.

[Lorenzen's letter to Beth of August 17th, 1959 (German text – see workshop website)]

And here is the English translation.

Highly esteemed, dear Mr. Beth,

Maybe this letter will reach you before you will have departed for Warsaw. I'm much looking forward to meet you there: for I just got your new voluminous book and am now reading in it with the greatest interest.



Darf ich das einmal kurz an Ihrem Beispiel 'festmachen'? Der Proponent P behauptet die logische Implikation $(x)[P(x) \to \overline{M(x)}] \wedge (Ey)[S(y) \& M(y)] \to (Ez)[S(z) \& \overline{P(z)}]$ d.h. er ist verpflichtet, die Konklusion zu behaupten, wenn sein Opponent die Prämissen behauptet

	Opponent	Proponent
(1)	$(x)[P(x) \to \overline{M(x)}]$	
(2)	$(Ey)[S(y) \& M(y)]$	$(Ez)[S(z) \& \overline{P(z)}]$

Fragment of a letter from Lorenzen to Beth, August 17, 1959

It is so encompassing that I have of course read just a part of it – but enough to now compliment you, without any reservations, on the mathematical elegance displayed in your proofs of all important theorems – and also on the new light thrown upon philosophical and historical connections.

The following paragraph clearly shows that Lorenzen was aware of the connection between Beth's semantic tableaux and his own investigations of the use of logical particles in dialogues:

Your new device, the semantic tableaux, is now very nicely and clearly expounded. There is also another reason for me to be particularly interested in these tableaux – and I would be very pleased if we could discuss this in detail sometime in Warsaw. When trying to define the term “definite,” which is used in my “*Einführung in die operative Logik and Mathematik* [Introduction to Operative Logic and Mathematics],” it occurred to me to investigate more closely how logical particles are used when they appear in a dialogue (between a proponent P and an opponent O). If one defines the way to make use of the logical particles in an obvious way, and if one then writes out the dialogues, then – with inessential transpositions – exactly your tableaux make their appearance.

In the example that follows Lorenzen used the same sequent (premises and conclusion) as Beth did in one of his examples in his book that had just appeared (Beth, 1959): the EIO-II syllogistic form Festino.

May I illustrate this just briefly, using your example “festino”? Let the proponent P assert the logical implication (x)[P(x)→¬M(x)]∧(Ey)[S(y)∧M(y)]→(Ez)[S(z)∧¬P(z)], i.e. he is obligated to assert the conclusion when his opponent asserts the premises.

opponent	proponent
(1) (x)[P(x)→¬M(x)]	
(2) (Ey)[S(y)∧M(y)]	(Ez)[S(z)∧¬P(z)]

For any assertion, one may always be asked to provide a “proof.” If O demands a proof for the assertion P(2), P may, however, first demand a proof for O(1), O(2). A “proof” for O(2) requires the specification of an element *a*

(3) S(a)∧M(a)	?

A “proof” for a conjunction requires that both conjuncts be asserted

(4) S(a)	
(5) M(a)	

Since also O(1) has been asserted, P may select any element, for instance a, so that O will then have to specify his assertion, which starts with (x), with respect to a

(5)	? (a)
(6) P(a) →¬M(a)	

Now P “proves” his assertion P(2) by

(6)	S(a)∧¬P(a)
(7)?	S(a)
(8)	¬P(a)

O can now not go on casting doubt on P(7), i.e. S(a), because he has asserted it himself before. When O wants to cast doubt on P(8), then he should, since that is a negation, assert himself P(a)

(9) P(a)	

But then P may also assert P(a)

(9)	P(a)

and O will now, because of (6), have to assert ¬M(a) as well

(10) ¬M(a)	

P may cast doubt on this assertion by asserting M(a) himself

	M(a)

on which O is not allowed to cast doubt, having at (5) already asserted it himself. Thus assertion O(9) has been refuted – and P and his assertion have "won."

Lorenzen then points out that logical validity may be defined dialogically, that is in terms of winning strategies:

> Now one may define a formula to be "logically valid," if there exists for it a winning strategy in this dialogue game (for elementary statements it is agreed that O may assert every elementary statement and P only those that were asserted by O before).

He also formulates a completeness (equivalence) theorem and is clearly aware that modification of the rules can yield another logic.

> The existence of a winning strategy is equivalent with the existence of a closed tableau – and thus with deducibility in an appropriate logical calculus (indeed here in the intuitionistic calculus, to get the classical calculus one should somewhat modify the rules of dialogue, for instance, so that one may always add $A \vee \neg A$).
>
> By non-finitary means, one will – I suspect – in the wake of your completeness proof be able to prove that for each formula there exists either a winning strategy for P or one for O (i.e. a counterexample model).
>
> Consequently, it seems to me that the tableaus might be helpful to establish a good connection between the "semantic" and the "operative" view –which matter we may perhaps discuss in Warsaw.
>
> With kindest regards – and again my congratulations on the completion of your book –
> always faithfully yours
> P. Lorenzen

Lorenzen's early papers did not make it easy to start working on completeness of systems. That was because in these papers there were as yet no systems that were clearly formulated. But later
much clearer formulations came forward. So what was the problem. Why so slow?

Here is list of reasons why:

1 As said, there were at first no clearly defined dialogue systems.

2 And early systems changed very often for there were permanently discussions about the best rules, so what system would one choose to investigate?

3 As time passed by there rose a suspicion that this had already been proved by someone, at least essentially, and that all that remained to be done would be placing some comments and minor corrections. That's not so sexy.

4 Some studies presenting elaborate mathematical reconstructions of some systems made one doubt whether they were talking about the same thing as the original authors. Remember Goethe's saying:

Die Mathematiker sind eine Art Franzosen: redet man zu ihnen, so übersetzen sie es in ihre Sprache, and dann ist es alsobald ganz etwas anders. [Mathematicians are a kind of Frenchmen: If one speaks to them, they will translate it into their language and then it is in no time something totally different.]

5 In the sixties and seventies there was a rather slow publication climate. At the universities, the important thing those days was not to publish or take a degree, but rather to teach and have discussions with students about social changes that would lead to a better society.

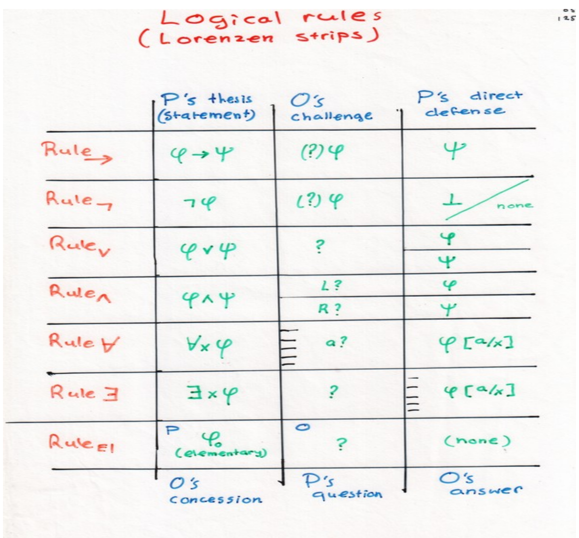
6 Moreover a proof would have to consist of many cases and would be too long and boring to be published in a journal.

7 If published in a book a proof may get lost between many other things that are also treated in the book, and thus escape notice.

8 Finally once proofs get published their reception will be hampered or even ruined by numerous misprints, not to mention the author's own slips of the pen.

Back to the early days. What one needs for completeness proofs is formal dialogue systems on a par with formal systems in other branches of logic, with which they may be compared. But the early dialogues were not "formal" in every sense. The language they used was supposed to be not an non-interpreted formalism but a meaningful language, and the meanings of elementary statements could influence the dialogues. Indeed, from the start, dialogue theory has been concerned primarily with *material dialogues,* not *formal dialogues.* This made it harder to compare the dialogue systems with other logical systems. Yet Lorenzen thinks (already in 1958) that he is on the way to justify, not only intuitionistic (or constructive) logic. And not only that, but also classical logic. For, as we saw in the letter to Beth, right from the start Lorenzen was aware that a little change in the proceedings would change the logic that resulted from the dialogue set-up.
Lorenzen's lecture in Warsaw gave us logical rules for the common logical particles, but not yet the procedural rules needed to complete a system. For altogether one needs three kinds of rules.

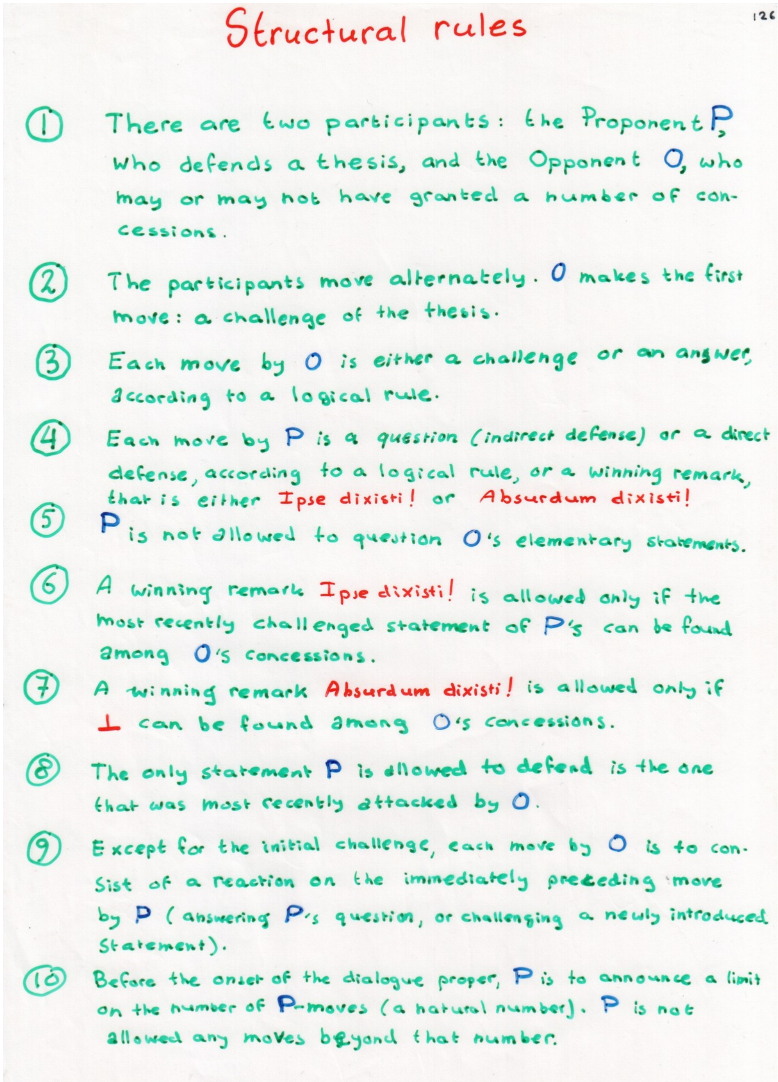
1 Logical rules


Logical rules
(Lorenzen strips)

	P's thesis (statement)	O's challenge	P's direct defense
$\text{Rule}_{\rightarrow}$	$\varphi \rightarrow \psi$	$(?)\varphi$	ψ
$\text{Rule}_{\neg}$	$\neg\varphi$	$(?)\varphi$	$\perp$ / none
$\text{Rule}_{\vee}$	$\varphi \vee \psi$	?	φ ψ
$\text{Rule}_{\wedge}$	$\varphi \wedge \psi$	L? R?	φ ψ
Rule $\forall$	$\forall x \varphi$	a?	$\varphi[a/x]$
Rule $\exists$	$\exists x \varphi$	?	$\varphi[a/x]$
Rule_{El}	P φ_0 (elementary)	O ?	(none)
	O's concession	P's question	O's answer

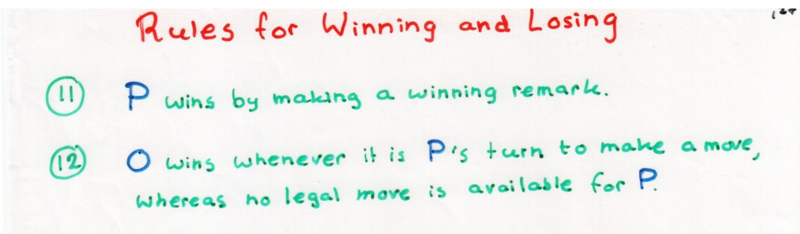
On the slide I show a version of the familiar logical rules. Notice that “attack” is replaced by “challenge” (for O, the opponent) or “question” (for P, the proponent), and that “defense” is replaced by “answer” (for O only)). There is rule that allows O to challenge elementary (atomic) formulas, but no rule that allows P to question them. For negation there are two rules possible, one with ⊥ as a defense or answer and one with no defense or answer.

2. Structural rules



The rules on this slide pertain to a system with “winning remarks” as in Barth & Krabbe (1982): *ipse dixisti!* (You said so yourself!) and *absurdum dixisti!* (You said something absurd!). The first three rules are part of most systems and will not be repeated when we get to discuss particular variants.

3. Rules for winning and losing. For example:



Kun Lorenz (dissertation 1961) was, I think, the first to formulate full systems with all three kinds of rules. That was a huge advance. Yet his choice of procedures made it pretty hard to show these systems complete. They give us the so-called D-dialogues (see Felscher, 1986, 344-345). D-dialogues are the toughest subject in the area.

STRUCTURAL RULES FOR D-DIALOGUES

1 No attacks on elementary formulas are allowed.
2 The initial thesis must be composite.
3. (Basic Rule) *P* may assert an elementary formula only if it was stated by *O* before.
4 *O* may attack each formula of *P*'s at most once. There are no such restrictions for *P*.
5 A defense move must answer the latest attack by the adversary that has not yet been answered

RULE FOR WINNING AND LOSING IN D-DIALOGUES

6 If it is party *N*'s turn to make a move, and no move is permitted by the system, then *N* has lost and its adversary has won the dialogue.

Lorenz's Basic Rule (also known as "Formal Rule") was adopted by Lorenzen in the system, (still not specified in every detail) that can be found in, or reconstructed from, his book *Metamathematik* (1962). Most likely, this was an E-system (see Felscher).

An E-system is defined by Felscher (1986, p. 345) as a D-system with one extra rule putting restrictions on the behavior of the Opponent (the E-rule):

STRUCTURAL RULES FOR E-DIALOGUES

The Rules for D-dialogues hold also for E-dialogues:

1 No attacks on elementary formulas are allowed.
2 The initial thesis must be composite.
3. (Basic Rule) *P* may assert an elementary formula only if it was stated by *O* before.
4 *O* may attack each formula of *P*'s at most once. There are no such restrictions for *P*.
5 A defense move must answer the latest attack by the adversary that has not yet been answered

But now we add the so-called E-Rule:

6 (E-Rule) After the first move (*O*'s attack on the initial thesis), each further move by *O* consists of a reaction on the immediately preceding move by *P*.

The RULE FOR WINNING AND LOSING IN E-DIALOGUES is also the same as in D-Dialogues:

7 If it is party *N*'s turn to make a move, and no move is permitted by the system, then *N* has lost and its adversary has won the dialogue.

I suspect that *Metamathematik* is the only place where Lorenzen (probably) proposes an E-system. For in later proposals for purely formal dialogue systems, Lorenzen seems to abandon the Basic Rule (and then the system is no longer an E-system in Felscher's sense). Yet these later systems are of the E-family in that they all have the E-rule. These are the purely formal systems proposed for constructive logic in Kamlah&Lorenzen (1967) *Logische Propädeutik,* in the appendix added to the third edition of Lorenzen's *Formale Logik* (1967), and in his *Normative Logic and Ethics* (1969). They are basically the same, though the system of *Formale Logik* slightly differs from the other two. Let us call them E_L-systems. After having introduced them, Lorenzen does not introduce any other purely formal systems, but rather speaks of formal strategies in material dialogue systems.

Let us have a closer look at the constructive E_L-systems. In these systems O may attack P's elementary formulas and there is no defense to such an attack. An exception is the system in *Formale Logik,* where P may defend by attacking the very same elementary put forward by O and thus win the game. Otherwise P cannot attack O's elementary formulas.
The structural rules of these games are as follows:

STRUCTURAL RULES FOR E_L-DIALOGUES

1 *P* may only either attack one of the composite formulas put forward by *O*, or defend himself against *O*'s last attack.

2 (E-rule) After the first move (*O*'s attack on the initial thesis), each further move by *O* consists of a reaction on the immediately preceding move by *P*.

Note that the E-rule excludes repetitive behavior by O. P may repeat attacks, but not defenses, for after the first defense against an attack, say the i-th attack by O, O must according to the E-rule attack P's defense and that will be the i+1-th attack by O, so P cannot return to a defense on the i-th attack (Rule 1).

We have the following rule for winning and losing:

RULE FOR WINNING AND LOSING IN E_L -DIALOGUES

3 *P* wins if he has to defend an elementary formula after *O*'s bringing forward of an identical elementary formula.

(In *Formale Logik,* we saw that *P* may attack an elementary formula, if he has to defend an identical formula and wins in that way.)

There is no stipulation about how O could win, probably when P can no longer make a legal move (that will not happen soon).

The rule for winning and losing could be interpreted to say that in order for P to win, O must first state some elementary formula q, and then later P states q and then O attacks it. This is, however, not the only way P could win according to the rule. It could also be that P first states q (the Basic Rule is not in force) and O attacks it, and P having no direct defense attacks other formulas of O and thus forces O to state q, while he still has to defend q himself.

That this is the right interpretation is borne out by the example Kamlah and Lorenzen provide on p.223 (ed. 1973) and some other examples in *Formale Logik* confirm that the Basic Rule does not hold and that the initial thesis may be elementary (pp. 165, 167, 168, ed. 1970). Therefore Lorenzen's last and best systems of purely formal dialectic are not E-systems, but only closely related to them. In fact they are equivalent to them (leaving aside the case of an elementary thesis, which is not allowed in E-systems). In fact they are even more closely related to the E_i-systems proposed by Barth and Krabbe (1982) and given this name by me in my 1985 *Synthese* article "Formal Systems of Dialogue Rules" in order to connect with Felscher's terminology. The subscript "i" stands for *ipse dixisti!* (You said so yourself!).

These are the structural rules for constructive E_i -systems:

STRUCTURAL RULES FOR E_i-DIALOGUES

1 *P* may only either attack (according to a logical rule) one of the composite formulas put forward by *O*, or defend himself against *O*'s last attack (either according to a logical rule or make a winning remark *ipse dixisti!* or *absurdum dixisti!*).

2 A winning remark *ipse dixisti!* is allowed only if the formula *P* has to defend (his most recently attacked statement) can be found among the formulas stated by *O*.

3 A winning remark *absurdum dixisti!* is allowed only if the formula ⊥ can be found among the formulas stated by *O*.

4 (E-Rule) After the first move (*O*'s attack on the initial thesis), each further move by *O* consists of a reaction on the immediately preceding move by *P*.

RULES FOR WINNING AND LOSING IN E_i -DIALOGUES

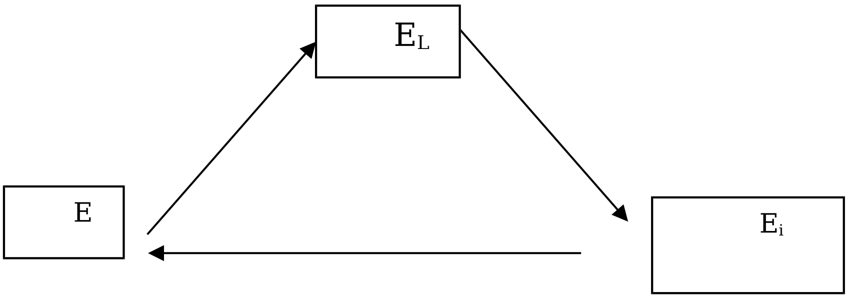
5 *P* wins by making a winning remark.

6 *O* wins if it is *P*'s turn to make a move, and no move is permitted by the system.

The equivalence of E_L-systems and E-systems now follows from the following theorem, which I call the Triangle Theorem:

THE E-TRIANGLE

E-Triangle Theorem: Let Z be a composite formula. Consider the constructive systems E_L, E_i, and E. Then, if there is a P-winning strategy for the thesis Z in one of these systems, there is one in all three of them.



Proof: (leaving out how to handle ⊥)
(1) From E_L to E_i. Trivial. Add *ipse dixisti* remarks.
(2) From E_i to E. See Krabbe (1985, *Synthese 63*) Section 3.1.
(3) From E to E_L. Trivial. Add attacks by O on P’s last asserted (elementary) formula

The mystery is why, although completeness is relatively easily proved for such systems as E_L or E_i, and the first of these were proposed in 1967, no generally accepted proof was put forward at the time. The appendix of the third edition of *Formale Logik* comes close, even though the formulation of the system is not clear about whether the E-rule is in force or not. It must, though, if we want to ascribe any relevance to the proof sketch that follows. But this proof fails to convince many by lack of explanations. Also it simply lacks a clear cut between two parts, one going from strategies to deductions and one going from deductions to strategies. What one may get convinced of when studying this proof is the second part: from deductions to strategies. But what is a proof? What is a proof for some may not be a proof for others.

So when in the seventies, at the University of Utrecht, Else Barth got me interested in dialogue logic, and our department had dialogue logic included in its teaching program, the need was felt to convince ourselves that the systems we taught were complete and we drew up, around 1973, some proofs for that purpose. Only part of the proofs were included in the courses, but the completeness theorem was presented to student as a fact, and so we had to be sure of it. These proofs were not published, for reasons stated above, and also because they were only meant to convince ourselves. This is the origin of the proofs that finally appeared in Barth and Krabbe (1982) and those for predicate logic in my dissertation of the same year (Krabbe, 1982), and finally those in Section 2 my paper of 1985. In *From Axiom to Dialogue* (1982) they were a kind of hidden among other matters in a full circle of equivalences:

FULL CIRCLE THEOREM

This theorem states the equivalence of there being for a sequent:

1. A closed dialogical tableau (winning strategy)
2. A closed deductive tableau
3. A natural deduction
4. An axiomatic derivation
5. No countermodel
6. A closed semantic tableau

Whereas all you needed for the completeness theorem of dialogue logic is the equivalence between there being a closed dialogical tableau and a closed deductive tableau. The rest is found in other literature.

This simplification: just going back and forth between two things was effected in my papers of 1985 and 1988. To go from strategies to deductions I transformed the more intuitive and practical method of demonstration in *From Axiom to Dialogue* into a single tree induction (returning to work of 1973). To go from deduction to strategies (which was for implicational logic left to the reader as an exercise, Barth & Krabbe 1982, p. 198) I used tree induction as well of some of the methods used in 1982 to go from closed semantic tableaux back to winning strategies.

Of course, there are errata in the paper of 1985. I brought a sheet of them. They are especially painful in the proof for D-dialogues, which is much more complicated than that for E-dialogues, though simpler than Felscher's proof (I guess).

Coming to speak of Walter Felscher's proofs, I think his work is fine and that his proofs are convincing. Only the E-dialogues are not really the dialogues intended by Lorenzen (the E_L- dialogues), but the latter, too, have, now been shown to be equivalent to the others (see the Triangle Theorem).

A final remark: My quick proofs paper of 1988 was completely ruined by errata. It was printed from the wrong version and even had my first name misspelled (with “c” instead of “k”). Reason enough not to read it. Unless you can get a corrected copy and want to use it as an introduction to Section 2 of the 1985 paper.

Thus the query for completeness finally tumbles into the quagmire of errata.

Thank you.

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